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A linearization approach for improving the computational efficiency of water distribution system ranking-based optimization algorithms

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Abstract

This paper presents a linearization approach to be used within the framework of water distribution system optimization problems where the searching algorithm is guided by the ranking of the possible solutions characterized by a system performance indicator such as the resilience index. The results obtained in addressing two different problems applied to real water distribution systems, namely, identification of the optimal placement of flow meters in a district meter area and multi-objective design of a water distribution system, show that a linearized hydraulic simulator although not capable of correctly describing the hydraulic functioning of the network, nonetheless enables each solution it finds to be assigned a value of the performance indicator whose ranking closely approximates the ranking of the corresponding solution that would be obtained if the full hydraulic simulator was used. This ensures significant reductions in computational times.

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Nomenclature

A	topological matrix
$\mathbf{A}_{10}$	topological incidence submatrix associated with the nodes with an imposed head

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$\mathbf{A_{11}}$	friction losses diagonal matrix
$\mathbf{A_{12}}$	topological incidence submatrix associated with the nodes with an unknown head
D	diameter of the pipe
$\mathbf{H_0}$	vector of known (imposed) nodal heads
$\mathbf{H}$	vector of unknown nodal heads
Ir	resilience index of the system
L	length of the pipe
$\mathbf{N}$	diagonal matrix whose elements on the diagonal are equal to n
n	discharge exponent in the formula used to represent the friction losses
n_0	number of nodes with known (imposed) head
n_{conn}	number of connecting pipes between districts
n_{fm}	number of flow meters placed in the connecting pipes between districts
n_n	number of nodes with an unknown head
n_p	number of pipes
n_{sim}	number of simulations
$\mathbf{Q}$	vector of unknown pipe discharges
$\mathbf{q}$	vector of nodal water demands
r	correlation coefficient
λ	Colebrook and White resistance coefficient
Ω	cross sectional area of the pipe

1. Introduction

In the last decades several studies have been focused on to the development of efficient and robust hydraulic simulation models of water distribution systems (Todini and Rossman, 2013). These models enable for an accurate characterization of the hydraulic functioning of the system providing pressure values for each node of the network and discharge and velocity values for each pipe.

These models are thus used within the framework of various procedures aimed at solving problems related to design, verification, renewal and optimal management of the water distribution system (Walski et al., 2003). In particular, very often, they are used to quantify compact indicators of system performance, such as, for example, the resilience index (Todini, 2000). These, in turn, are used as objective functions in the context of optimization problems (Raad et al., 2010), such as the multi-objective design of water distribution systems (e.g. Prasad and Park, 2004; Farmani et al., 2005, Creaco and Franchini, 2012) or the design of district meter areas (e.g. Di Nardo et al., 2011, Alvisi and Franchini, 2014a).

In the former case, the design problem is typically formulated as a two-objective problem in which one looks for the optimal solution in terms of diameters to be attributed to the network pipes in order to minimize costs and maximize system reliability: the costs depend on the diameters chosen and are quantified without relying on the hydraulic simulator; reliability, on the other hand, is quantified by means of a suitable compact indicator (for example the resilience index) calculated on the basis of the nodal pressures obtained by hydraulic simulation.

In the latter case, the problem entails first dividing up the nodes in some way in order to form the district meter areas (see, for example, Alvisi and Franchini, 2014a) and then identifying, between one district and another, which connecting pipes should be closed and which should be left open and fitted with a flow meter. During the search process, for a given allocation of nodes among the district meter areas, and hence for assigned connecting pipes and a pre-established number of meters to be installed, it is necessary to consider all the different combinations of open and closed connecting pipes and run hydraulic simulations to identify the most advantageous one, i.e. the combination that maximizes an appropriate indicator, which in this case as well could be the resilience index (Di Nardo et al., 2011). Again, the hydraulic simulator is used to quantify a compact indicator which is useful for comparing different solutions to the problem considered.

Nowadays, the computational resources offered by processors and hydraulic simulators of water distribution systems, which are often based on the efficient global gradient method proposed by Todini and Pilati (1988), enable us to run hydraulic simulations and calculate pipe discharges and nodal heads with computational times that are generally very short (Giustolisi et al., 2012). However, it is worth noting that when optimization problems are applied to real networks (characterized by hundreds or thousands of nodes and pipes), these computational times tend to increase, both because of the large number of hydraulic simulations that need to be carried out and because of the complexity of the individual simulations. In other words, in the case of real networks, due to the lengthening of simulation times combined with the need to run a large number of hydraulic simulations, the computational burden can come to represent a practical limit to the application of an optimization procedure. This is demonstrated by the fact that recently, again in connection with the problems mentioned earlier, researchers have investigated different approaches specifically aimed at limiting this computational burden. For example, in the context of the multi-objective design of water distribution systems, techniques have been investigated which are based on reducing the search domain (Creaco and Franchini, 2012), using efficient hybrid optimization algorithms (Wang et al., 2012) or, within the realm of optimization algorithms of genetic type, using operators which enable the optimizer to be "heuristically" directed towards an appropriate direction so as to reduce the number of simulations that need to be run (Barlow and Tanyimboh, 2012; Pelourdeau et al., 2012). Analogously, in the case of district meter areas, heuristic approaches have been proposed to limit the number of combinations of open and closed pipes to be considered and thereby also limit the number of hydraulic simulations to be run (Alvisi and Franchini, 2014a).

In reality, it is important to stress that in both of the problems mentioned earlier by way of example, the ultimate aim of hydraulic simulations of a network is that to be able to rank the different solutions hypothesized in order to identify the best one or ones.

Based on these considerations we present the application of an approach, originally proposed in Alvisi and Franchini (2014b) and based on replacing the standard hydraulic simulator with a "simplified" one to be used within the framework of ranking-based optimization approaches. The simplified simulator, though not capable of correctly characterizing the hydraulic functioning of the network, it would nonetheless allow us to associate a value of the performance indicator with each solution so as to reproduce, in a sufficiently approximate manner, the ranking/ordering of the corresponding solutions we would obtain if we used the correct hydraulic simulator, while at the same time achieving significant reductions in the computational times.

The paper is organized as follow: after briefly illustrating the global gradient method (Todini and Pilati, 1988) for solving looped networks, the structure of the simplified simulator is described. The application of the proposed approach in procedures for the optimal placement of flow meters in a district meter area and the multi-objective design of a water distribution system for real water distribution systems is presented and the results obtained are discussed; finally, conclusions are presented.

2. The linearized hydraulic simulator

According to the demand-driven formulation proposed by Todini and Pilati (1988), the hydraulic simulation model of a generic network made up of n_p pipes, n_n nodes with an unknown head and n_0 nodes with an imposed head can be written in the following manner:

$$\begin{bmatrix} \mathbf{A_{11}} & \mathbf{A_{12}} \\ \mathbf{A_{21}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{Q} \\ \mathbf{H} \end{bmatrix} = \begin{bmatrix} \mathbf{-A_{10}H_0} \\ \mathbf{q} \end{bmatrix} \tag{1}$$

where $\mathbf{Q^T} = \left[Q_1, Q_2, ..., Q_{n_p} \right]$ represents the vector of unknown pipe discharges, $\mathbf{H^T} = \left[H_1, H_2, ..., H_{n_n} \right]$ the vector of unknown nodal heads , $\mathbf{H_o}^\mathbf{T} = \left[H_{01}, H_{02}, ..., H_{0n_0} \right]$ the vector of known nodal heads and $\mathbf{q^T} = \left[q_1, q_2, ..., q_{n_n} \right]$ the vector of nodal water demands.

Matrix $\mathbf{A_{11}}$ is a diagonal matrix ($n_p \times n_p$) whose elements other than zero, taking into account only friction losses, can be expressed in the following manner:

$$\mathbf{A_{11}}(i,i) = \alpha_i \left|Q_i\right|^{n-1} \quad i = 1 : n_p \tag{2}$$

where α_i represents the resistance coefficient of the i-th pipe as a function of roughness, diameter and length, while n represents the discharge exponent in the formula used to represent the friction losses and takes on a value close or equal to 2 under the conditions of turbulent motion typically present in water distribution systems.

Considering, for example, the Darcy (1858)-Weisbach (1845) equation:

$$\Delta H = \frac{\lambda}{D}\frac{v^2}{2g}L = \frac{\lambda}{D}\frac{\left(Q/\Omega\right)^n}{2g}L \ \text{ with } n=2 \tag{3}$$

we would have:

$$\alpha_i = \frac{\lambda_i L_i}{2gD_i\Omega_i^n} \quad \text{with} \quad n=2 \tag{4}$$

where D_i, Ω_i and L_i are respectively the diameter, cross sectional area and length of the i-th pipe and λ_i is the associated resistance coefficient (Colebrook and White, 1937).

The matrices $\mathbf{A_{12}}$ ($n_p \times n_n$) and $\mathbf{A_{10}}$ ($n_p \times n_0$) are the topological incidence submatrices associated, respectively, with the nodes with an unknown head and nodes with an imposed head; these matrices are derived from the topological matrix $\mathbf{A} = \left[\mathbf{A_{12}} \vdots \mathbf{A_{10}}\right]$ ($n_p \times (n_n + n_0)$) (Todini and Pilati, 1988), in which the element $\mathbf{A}(i, j)$ can take on the values 0, -1 and 1: $\mathbf{A}(i, j) = 0$ if the i-th pipe does not have the j-th node at one end; if the i-th pipe has the j-th node at one end, $\mathbf{A}(i, j) = 1$ or $\mathbf{A}(i, j) = -1$ depending, respectively, on whether the flow that is hypothesized (wholly arbitrarily) in the i-th pipe enters or exits through the j-th node.

Finally, the matrix $\mathbf{A_{21}}$ ($n_l \times n_p$) is the transpose of $\mathbf{A_{12}}$.

The system of equations (1) is not linear, since the matrix $\mathbf{A_{11}}$ contains the unknown $\mathbf{Q}$ raised to the power n-1 (see eq.(2)). This system can thus be solved iteratively relying on the following system of equations which gives the values of $\mathbf{Q}$ and $\mathbf{H}$ at the generic step $k+1$, the values at step k being known (Todini and Rossman, 2013, Franchini and Alvisi, 2010):

$$\begin{aligned} \mathbf{H^{k+1}} &= \left(\mathbf{A_{21}G^{-1}A_{12}}\right)^{-1}\left[\mathbf{A_{21}G^{-1}}\left(\left(\mathbf{G}-\mathbf{A_{11}}\right)\mathbf{Q^k}-\mathbf{A_{10}H_0}\right)-\mathbf{q}\right] \\ \mathbf{Q^{k+1}} &= \mathbf{Q^k}-\mathbf{G^{-1}}\left(\mathbf{A_{11}Q^k}+\mathbf{A_{12}H^{k+1}}+\mathbf{A_{10}H_0}\right) \end{aligned} \tag{5}$$

where $\mathbf{G=NA_{11}}$ and $\mathbf{N}$ is a diagonal matrix ($n_p \times n_p$) whose elements on the diagonal are equal to n (discharge exponent in the friction loss formula).

Now, going back to the system of equations (1), it is evident that if we assumed $n = 1$, that is, if we linearized the energy balance equation (see eq.(3)), from a hydraulic viewpoint we would arrive at an approximate hydraulic characterization of the network; it is important to observe that in such a case the system would be made up only of linear equations and its solution would be immediately given by:

$$\begin{bmatrix} \mathbf{Q} \\ \mathbf{H} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{-A}_{10}\mathbf{H}_0 \\ \mathbf{q} \end{bmatrix} \tag{6}$$

Practically speaking, assuming n = 1 enables us to arrive at the formulation of a *linearized* hydraulic simulation model (see also eq.(2)) capable of approximately characterizing the network's behaviour while significantly reducing the computational burden.

This approach can clearly not be used for an accurate hydraulic verification of the functioning of a network; however, as previously noted, in the case of optimization problems solved by means of ranking-based algorithms, what counts is not so much the value per se of an individual performance indicator, but rather the fact that the generic performance indicator is better or worse than that of another solution (and the solutions are ranked accordingly). Moreover, a linearized hydraulic simulation model introduces the same type of approximation in all solutions; it is thus reasonable to expect that the values of the performance indicators connected to the solutions produced by them, though approximate in nature, will have a ranking which closely matches that of the corresponding performance indicators calculated on the basis of correct (but computationally much more cumbersome) hydraulic simulations.

In the following applications the limits and advantages of using a linearized hydraulic simulator in place of correct one to address the two ranking-based optimization problems previously mentioned are evaluated.

3. Applications

3.1. Case studies

The proposed approach was used within the framework of the optimal placement of flow meters in a district meter area and the multi-objective design problem applied to two real water distribution systems in northern Italy. The two network, designated as Net-A and Net-B, are represented, respectively, in Fig. 1 and Fig. 2: network Net-A was used in addressing the problem of optimal placement of flow meters and network Net-B in addressing problem of optimal design.

More in details, network Net-A extends over a total distance of about 160 km, is fed from a tank located in proximity to the town centre which ensures a head of about 40 m relative to the ground level and serves around 21000 inhabitants. The corresponding hydraulic model consists of n_n=413 nodes with an unknown head, n_0 =1 node with an imposed head and n_p=465 pipes. Network Net-A was selected since, as already mentioned earlier and clearly explained in Alvisi and Franchini (2014a), the problem of creating district meter areas can be structured in two steps: the first involves dividing up the network nodes so as to form the district meter areas and the second consists in identifying which connecting pipes between one district and another should be closed and which should be left open and fitted with a flow meter. The network considered here is exactly the one used by Alvisi and Franchini (2014a) as an example for determining the optimal placement of flow meters and therefore a hypothetical division of the nodes into two district meter areas is already available (step 1): this division is shown in Fig. 1. Given this hypothetical scheme, we have n_{conn}=13 connecting pipes between the two districts. Assuming that n_{fm} flow meters are placed in the connecting pipes and the remaining n_{conn}-n_{fm} pipes are closed off, the optimal solution, in terms of which pipes to leave open and fit with a flow meter and which to close, can be identified, as suggested by Di Nardo et al. (2011) (see also Alvisi and Franchini, 2014a), as the one which maximizes the resilience index Ir of the system (Todini, 2000), defined as:

$$Ir = \frac{\sum_{i=1}^{n_n} q_i \left(H_i - H_i^* \right)}{\sum_{m=1}^{n_0} Q_m H_m - \sum_{i=1}^{n_n} q_i H_i^*} \tag{7}$$

where q_i, H_i and H_i^* are respectively the water demand, available head and minimum required head at the i-th node, and Q_m and H_m are the discharge and head associated with the m-th node having an imposed head. With specific reference to this case study, a minimum required head H_i^* of 25 m was assumed for all nodes. From a practical standpoint, the optimal solution can thus be identified by considering all of the possible configurations of n_{fm} open connecting pipes (and, accordingly, the n_{conn}-n_{fm} closed connecting pipes), carrying out a hydraulic simulation for each in order to associate a corresponding value of the resilience index and selecting the solution which provides the greatest value. For example, in the case of a number of flow meters n_{fm} equal to 1 or 2 or 3, the overall number of hydraulic simulations would be equal to n_{sim}= 13+78+286= 377.

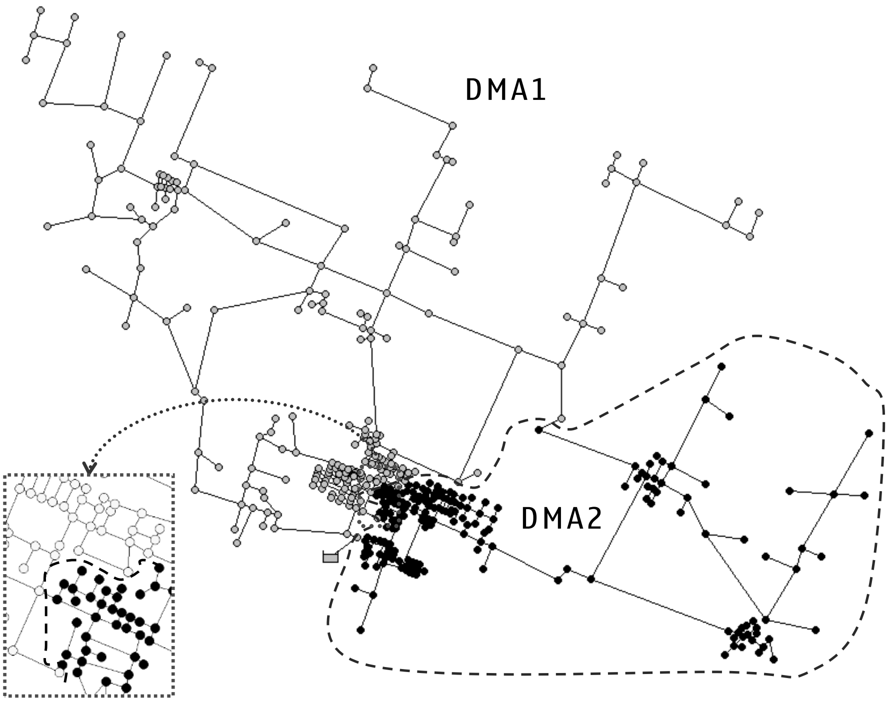

Fig. 1. Water distribution network Net-A. In the figure, the nodes belonging to each of the two districts the network is assumed to be divided into are shown in light and dark shades.

Network Net-B extends over a total distance of around 90 km, is fed by two tanks which ensure an average head of about 30 m and serves about 65000 inhabitants. The corresponding hydraulic model is made up of n_n=536 nodes with an unknown head, n_0 =2 nodes with an imposed head and n_p=825 pipes. The possible values of the design diameters and respective costs are shown in Tab. 1.

An optimal design for this network was sought with the aim of minimizing costs and maximizing reliability, quantified through the resilience index Ir (see eq.(7)), assuming a minimum required head H_i^* of 25 m for all nodes. The search for the optimal solutions of the two-objective problem was conducted using the NSGA-II algorithm (Deb et al., 2000), assuming that each individual was characterized by a number of genes equal to the number n_p of pipes in the network, the diameter to be attributed to each pipe being encoded in the corresponding gene (Prasad and Park, 2004, see also Creaco and Franchini, 2012). The number of the possible diameters associable with each pipe is 11 (see Tab. 1). A population of 200 individuals (each with 825 genes) and a number of generations (new populations) equal to 500 were assumed at the optimization stage.

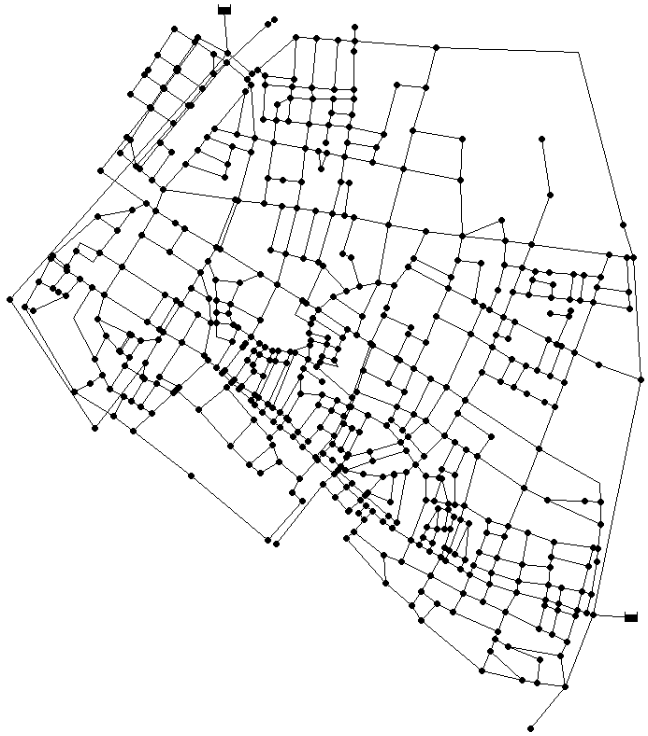

Fig. 2. Water distribution network Net-B.

Table 1. Network Net-B: design diameters and respective costs per unit of pipe length.

D [mm]	*cost [€/m]*
45	185
60	203
80	227
100	231
150	272
200	299
250	328
300	360
350	399
400	439
500	503

3.2. Results

The problem of optimal placement of flow meters in network Net-A was solved by using both the hydraulic simulator (see eq.(5)) and the linearized simulator (see eq.(6)); Fig. 3a shows a comparison between the resilience index values for each of the n_{sim} =377 possible solutions calculated with the two simulators. As may be observed, linearizing the model – that is, assuming an exponent of n=1 in the friction loss formula – clearly results in an inaccurate estimation of head losses and hence of nodal heads, so that in absolute terms the consequent values of *Ir* are very different from those obtained with the hydraulic simulation model, in which it is assumed that n=2. On the other hand, it is evident that a certain correlation exists between the resilience index values calculated for each solution using the two different approaches. This is even more evident if we consider Fig. 3b, which compares the rankings of the solutions defined on the basis of the resilience indices calculated respectively with the hydraulic simulator and linearized simulator. As can be seen, the points tend to form a very narrow cloud around the 45° diagonal, which represents a perfect match, thus showing that the ranking of the n_{sim} =377 possible solutions obtained with the linearized simulator very closely approximates the ranking obtained with the hydraulic simulator. The corresponding correlation coefficient r for the two rankings indeed takes on a value close to 0.99. As regards the computational times, it is worth observing that, using the same processor (Intel Celeron 2.4 GHz), the hydraulic simulation of the n_{sim} solutions took 70.2 sec., whereas 3.6 sec., about one twentieth of the time, were sufficient with the linearized simulator.

Practically speaking, with the linearized simulator it is possible to arrive rapidly at an identification of most of the best solutions in terms of flow meter placement in the network, so that the only task remaining for the hydraulic simulator will be to carry out an a posteriori verification of these solutions in order to correctly quantify all of the aspects related to the hydraulic functioning of the system, namely, nodal heads, pipe discharges and velocities, etc.

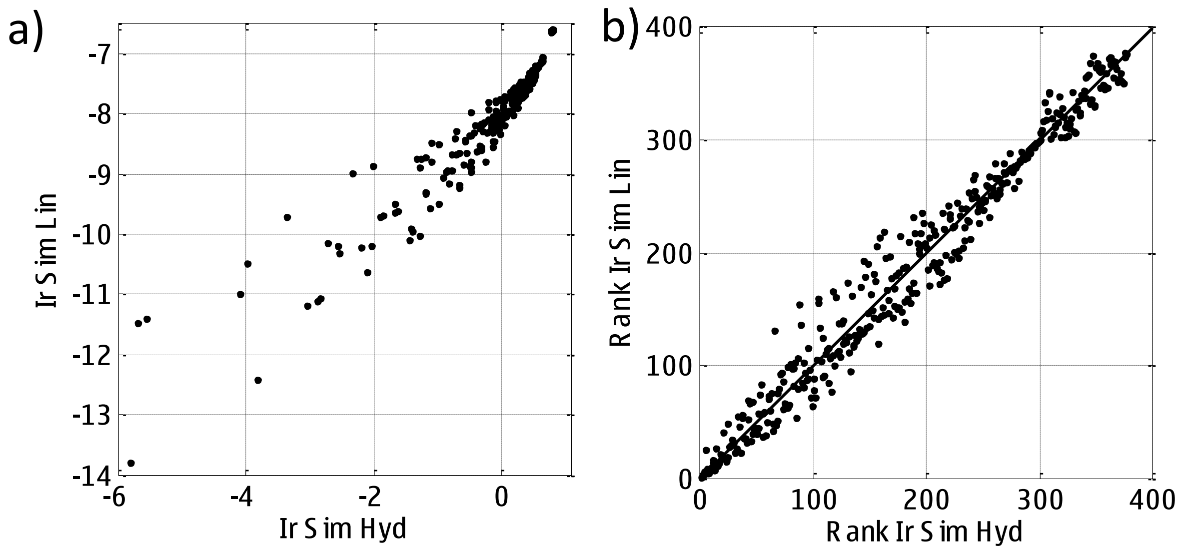

Fig. 3. Identification of the optimal placement of flow meters in the context of the problem of optimal district meter area design: a) comparison between the resilience index values for each solution for the placement of flow meters in the connecting pipes as calculated with the hydraulic simulator (n=2) (Ir Sim Hyd) and linearized hydraulic simulator (n=1) (Ir Sim Lin); b) comparison between the ranking of solutions obtained with the hydraulic simulator (n=2) and linearized hydraulic simulator (n=1).

With regard to the problem of finding an optimal design for network Net-B, the linearized simulator and hydraulic simulator were used within the framework of the multi-objective design procedure proposed by Prasad and Park (2004), which is based on use of the NSGA-II algorithm. Two distinct optimization processes were thus carried out: the first one, for quantifying the *Ir*, was based on use of the hydraulic simulator, and the second one on use of the linearized simulator. Fig. 4 compares the Pareto fronts obtained from the two optimization processes. In particular, as regards the front resulting from the optimization process in which the linearized simulator was used, the values of *Ir* shown in the figure are the ones obtained by evaluating a posteriori, with the hydraulic simulator, each of the solutions making up the final population provided by the optimization process based on use of the linearized simulator. As can be observed, given the same optimizer parameters (population size, number of generations, etc.), the linearized simulator-based optimization process leads to the identification of a Pareto front, and thus a set of optimal solutions, which is wholly equivalent, if indeed not better, than the one provided by the optimization process based on use of the hydraulic simulator. However, the computational times of the procedure using the linearized simulator, including post-analysis with the hydraulic simulator, are distinctly shorter than those of the procedure based directly on use of the hydraulic simulator: about 28 minutes in the former case and 809 minutes in the latter (0.5 h vs. 13.5 h).

For all practical purposes, therefore, the linearized simulator enables us to define values of the system performance indicator for the different solutions and a consequent ranking able to guide the optimization process correctly while requiring distinctly shorter computational times than a hydraulic simulator. The optimization process may be up to dozens of times shorter, thus making the procedure reasonably applicable also in real, complex networks.

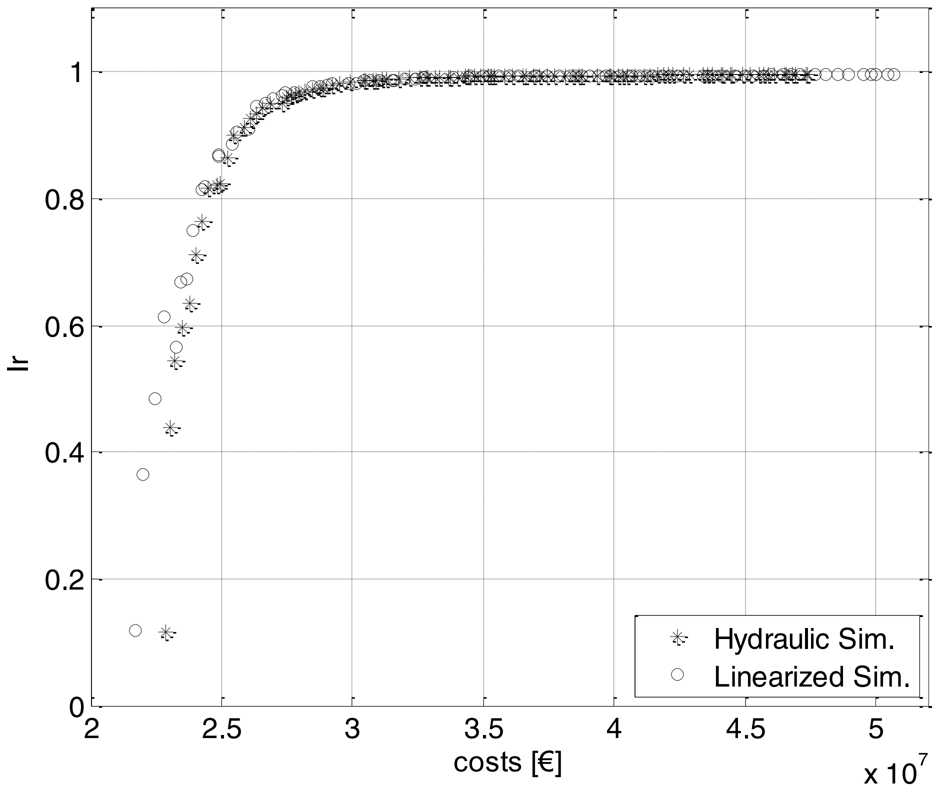

Fig. 4. Multi-objective network design: comparison between the Pareto front obtained using the hydraulic simulator (n=2) (+ symbol) and linearized hydraulic simulator (n=1) within the framework of the multi-objective optimization procedure and re-evaluating the solutions of the final population with the hydraulic simulator (n=2) *a posteriori* (o symbol).

4. Conclusions

The results obtained with respect both to the placement of flow meters and optimal design reveal that using a linearized hydraulic simulator is reasonable whenever an optimization process is "guided" by the ranking of the solutions considered, which are in turn evaluated by means of a performance indicator. In such situations, as far as the solution rankings are concerned, the approximations introduced when using the linearized simulator are minimal and in any case enable identification of the near-optimal solution(s) while significantly reducing computational times. The proposed approach can thus be considered a valid technique at least for "roughly" solving the problem, i.e. identifying a set of near-optimal solutions in a short space of time, even in the case of real networks, while there will always remain the possibility of analyzing these solutions via a hydraulic simulator in a post-processing phase, or of refining the optimization process by means of the hydraulic simulator starting from a very good initial set of solutions.

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